Causal Symmetry — Technical Supplement IV:
Spin, Detectability, and What a Null Excludes

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Abstract

This supplement is the last increment of the desk-development program: it connects the phase-coherent echo template of Supplements II–III to what a gravitational-wave detector could actually conclude. **First**, the echo delay is extended to spinning remnants, $\Delta t = (4GM/c^3)\,(1 + 1/\sqrt{1-a^2})\ln(M/M_{\rm Pl})$: $235 \to 275 \to 386\,$ms at $65\,M_\odot$ for $a = 0$, 0.67, 0.9. **Second**, the echo-train signal-to-noise ratio is computed against an analytic fit to the Advanced-LIGO design curve: the distance-independent ratio $\rho_{\rm echo}/\rho_{\rm rd}$ falls from 0.69 (near-mirror wall) to 0.14 ($\Gamma = 0.95$), so echoes at a $\rho_{\rm det} = 8$ threshold require ringdown SNRs of $\gtrsim$ 12–18 for $\Gamma = 0.1$–0.5 — out of reach for GW150914-class events, within reach for loud events in the A+ era. **Third**, the exclusion logic runs opposite to naive intuition: because a nearly-mirror wall rings loudest, a null search bounds Γ *from below*, $\Gamma \geq \Gamma_{\rm min}(\rho_{\rm rd})$ — deep nulls push away from the mirror-box (dimension-counting) corner of the Γ table and toward absorbing walls, degenerate with classical horizons. A detection, conversely, measures Γ twice over: through the train's decay rate and through the absorption of successive echoes. With this, every item on the program's tractable list is executed, estimated, or reclassified; the closing section states the frontier.

1 Scope

Supplements II–III produced a template; this supplement asks what an instrument can do with it. Everything here is forecast, not data analysis: no strain data are touched, and the noise model is an analytic stand-in. The two deliverables are the spin-corrected delay (Section 2) and the detectability/exclusion machinery (Sections 3–4).

2 Spin-corrected echo delay

For a Kerr horizon, $r_\pm = GM/c^2\,(1 \pm \sqrt{1-a^2})$, the near-horizon tortoise coordinate behaves as $r_* \simeq [(r_+^2 + (aGM/c^2)^2)/(r_+ - r_-)]\ln(r - r_+)$, and the round trip to structure at proper Planck depth gives, to leading logarithmic accuracy,

$$\Delta t_{\rm echo}(M,a) \;\simeq\; \frac{4GM}{c^3}\left(1 + \frac{1}{\sqrt{1-a^2}}\right)\ln\frac{M}{M_{\rm Pl}}, \tag{1}$$

reducing to the Schwarzschild $8GM\ln(M/M_{\rm Pl})/c^3$ at $a = 0$ and agreeing with the spin correction quoted in the echo-search literature [1]. At $65\,M_\odot$: $235\,$ms ($a = 0$), $275\,$ms ($a = 0.67$, GW150914-like), $386\,$ms ($a = 0.9$); see Figure 1. Spin therefore shifts the comb spacing $1/\Delta t$ from 4.3 to 2.6 Hz across this range — a measurable, template-level effect. (The barrier itself is still computed at $a = 0$; the Teukolsky upgrade is listed in Section 5.)

*Numerics reproducible from `detectability.py` (uses the Regge–Wheeler reflectivity table of Supplement III).

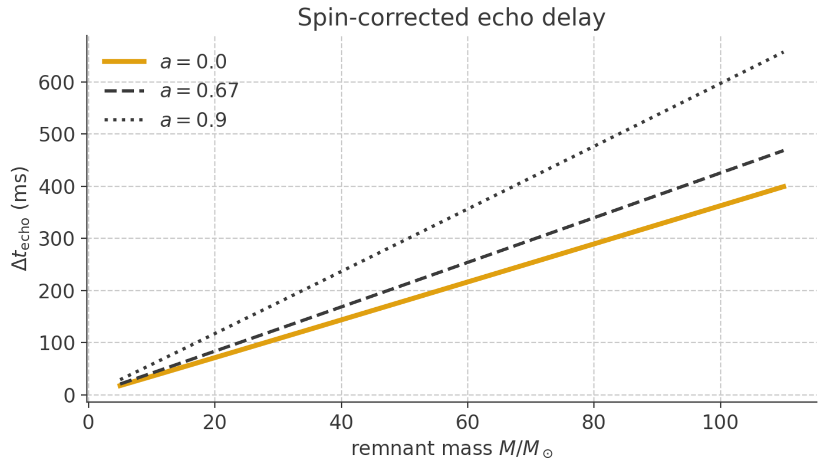

Figure 1: Echo delay versus mass for three spins, Eq. (1).

3 Detectability of the echo train

For a signal h in stationary Gaussian noise with one-sided spectral density $S_n(f)$, the matched-filter signal-to-noise ratio is $\rho^2 = 4\int |\tilde{h}(f)|^2/S_n(f)\,df$. We evaluate the ringdown $\rho_{\rm rd}$ and the echo train $\rho_{\rm echo}$ (template of Supplements II–III, numerical Regge–Wheeler barrier, 20–1024 Hz band) using a standard analytic fit to the Advanced-LIGO design sensitivity,

$$S_n(f) = 10^{-49}\left[x^{-4.14} - 5x^{-2} + \frac{111\,(1-x^2+x^4/2)}{1+x^2/2}\right]\ \mathrm{Hz}^{-1}, \qquad x = \frac{f}{215\,\mathrm{Hz}}, \tag{2}$$

valid above $\sim 20\,$Hz. *This fit is an approximation adopted for transparency; production analyses must substitute the official tabulated PSDs.* The useful quantity is the **distance-independent ratio** $\rho_{\rm echo}/\rho_{\rm rd}$ (Figure 2, left): it falls monotonically from 0.69 as $\Gamma \to 0$ (near-mirror wall, long train) to 0.14 at $\Gamma = 0.95$ (absorbing wall, dead cavity). Detectability then reduces to a required ringdown SNR, $\rho_{\rm rd} \geq \rho_{\rm det}/(\rho_{\rm echo}/\rho_{\rm rd})$:

Γ	$\rho_{\rm echo}/\rho_{\rm rd}$	required $\rho_{\rm rd}$ ($\rho_{\rm det} = 8$)	remark
0.10	0.65	12.4	near-mirror
0.25	0.57	13.9	Supplement II fiducial
0.50	0.45	17.6	strongly coupled

GW150914's ringdown-only SNR was of order 7–8, below all of these — consistent with the absence of a confident detection to date [1, 2, 3]. The forecast's content is positive, however: ringdown SNRs of $\gtrsim 14$ are expected for loud events at design sensitivity and routine in the A+/next-generation era, so *the template of this program is testable with announced instruments, no new technology required.*

4 What a null excludes — and what a detection measures

Because the ratio *decreases* with Γ, the exclusion logic runs opposite to naive intuition. A nearly-mirror wall ($\Gamma \to 0$) rings loudest; an absorbing wall rings quietest. A null result at ringdown SNR

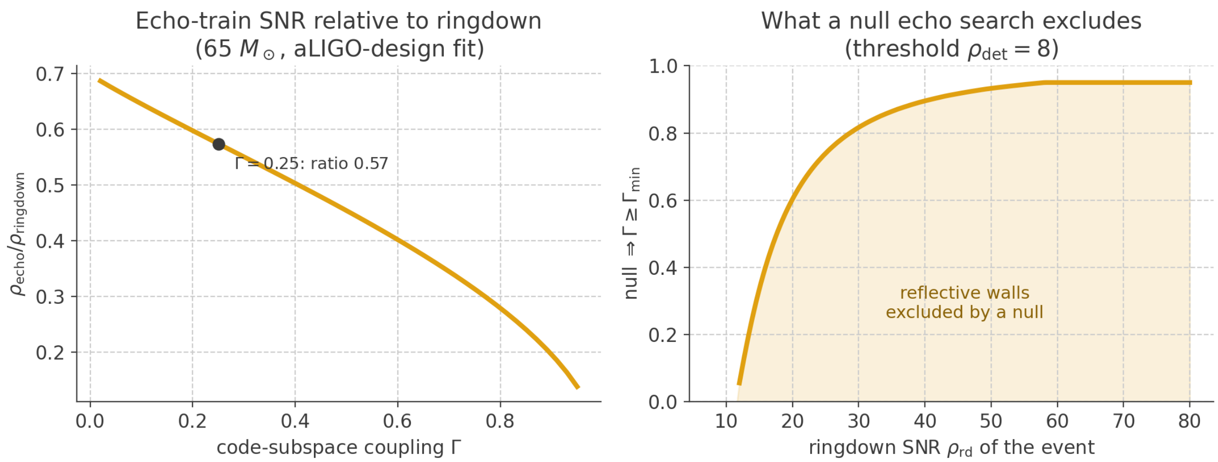

Figure 2: *Left:* echo-train SNR relative to ringdown SNR versus the code-subspace coupling Γ (65 $M_\odot$, PSD fit of Eq. (2)). *Right:* a null search at threshold $\rho_{\mathrm{det}} = 8$ excludes the shaded region: reflective walls, $\Gamma < \Gamma_{\mathrm{min}}(\rho_{\mathrm{rd}})$.

ρ_{rd} therefore excludes Γ *below* a bound,

$$\text{null} \;\Rightarrow\; \Gamma \;\geq\; \Gamma_{\mathrm{min}}(\rho_{\mathrm{rd}}), \tag{3}$$

shown in Figure 2 (right): $\Gamma_{\mathrm{min}} \approx 0.1$ at $\rho_{\mathrm{rd}} = 12.4$, 0.5 at 17.6, approaching 0.95 for $\rho_{\mathrm{rd}} \gtrsim 60$. Read against the Γ table of Supplement III, deep nulls push away from the mirror-box (dimension-counting) corner and toward strongly absorbing walls — which are observationally degenerate with classical horizons, since "no Planck-depth structure at all" also yields a null. Two honest consequences: (i) nulls can never single out Causal Symmetry's absorbing regime over ordinary general relativity; (ii) a *detection* would be doubly informative, measuring Γ through the train's geometric decay rate $|R_b R_w|$ and through the absorption imprint on successive echoes, while the delay and comb measure (M, a) independently of Γ. The framework's gravitational-wave sector is thus fully specified as a measurement problem: one function, $\Gamma(\omega)$, two independent handles on it, and an instrument era in which the required SNRs occur.

5 The frontier

With this supplement, every item on the program's tractable list has been executed, estimated, or honestly reclassified. What remains requires resources beyond desk development, and it is worth being precise about why:

1. **Strain-data analysis.** Fitting the template to real events requires the official PSDs, calibrated strain, and the LVK analysis stack (PyCBC/bilby); the forecast here deliberately stops at the boundary of what can be done without them.

2. **The Teukolsky barrier.** Spin enters the delay analytically (Eq. (1)) but the barrier reflectivity at $a \neq 0$ requires the Sasaki–Nakamura machinery and spheroidal eigenvalues — mechanical, but demanding validation against published tables.

3. **The spinfoam transfer matrix.** The decisive theoretical test (Supplement II, Section 2.4) needs Lorentzian EPRL amplitude numerics of the `sl2cfoam-next` class and, realistically, collaboration with the loop-quantum-gravity community.

4. **A first-principles $\Gamma(\omega)$.** The three candidate scalings span $e^{-10^{77}}$ to $O(1)$ (Supplement III); selecting among them is a quantum-gravity derivation no amount of phenomenology can substitute for.

5. **Peer engagement.** The consolidated manuscript v2.0 is the circulation document; items 3 and 4 are its natural collaboration hooks.

The desk program is complete. Its net yield: a principle turned into a three-clause boundary condition with exact tolerances; a template with mass-, spin-, and Γ-dependence; two observational branches with named signatures; and — the part most likely to earn the respect of referees — a framework that has published its own bad news at every step.

References

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