# Causal Symmetry — Technical Supplement III:
## The Barrier Made Exact, the Γ Problem, and the Page Fork

Felix J. Ruch*

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**Abstract**

Three further steps, including one unwelcome but clarifying result. **First**, the toy barrier of Supplement II is replaced by a numerical solution of the Regge–Wheeler equation ($s = 2$, $\ell = 2$): the potential peak reproduces the literature value to 0.3%, scattering unitarity holds to $10^{-13}$, and the echo template is essentially unchanged (first-echo amplitude 0.279 vs. 0.269) — the template is now production-grade up to spin. **Second**, the wall coupling $\Gamma$ is confronted at first principles. Three candidate scalings exist: dimension counting gives $\Gamma \sim e^{S_{\text{matter}} - S_{BH}} \approx e^{-10^{77}}$ at a solar mass — observationally dead; an $O(1)$ coupling requires a third, explicitly dynamical clause of the WHO condition ("funneling"); and a thermalized boundary reduces to the Boltzmann reflectivity of dissipative models. Observable flat-reflectivity echoes are therefore not a generic consequence of Causal Symmetry but a measurement of the funneling clause. **Third**, the Page-curve tension is resolved into a structural dichotomy aligned with the lifetime fork: on the early branch, cross-domain transfer *replaces* islands as the unitarity mechanism; on the late branch, islands purify the radiation and the Past Censor emits only a Planck-scale residue ("thin bridge"), which weakens — but does not destroy — the emergent-domain narrative. Energy bookkeeping is resolved by reframing (only information balance is frame-independent across disconnected domains), and the CMB channel is estimated at $2|\beta_k| \sim 10^{-6}$–$10^{-5}$, below current feature sensitivity. These results are folded into the consolidated manuscript v2.0.

# 1 Scope

Conventions follow Supplements I–II. This supplement closes, or honestly reclassifies, the remaining tractable items on the development ledger: the production barrier (Sec. 2), the first-principles wall coupling (Sec. 3), the Page tension (Sec. 4), energy bookkeeping (Sec. 5), and the cosmological channel (Sec. 6). Where a result is unfavorable to the framework's observability, it is stated as such; a research program earns credibility by publishing its own bad news.

# 2 The barrier made exact

Supplement II modeled the photon-sphere barrier with a smooth phenomenological reflectivity. Here the axial ($s = 2$, $\ell = 2$) Regge–Wheeler equation [1]

$$\frac{d^2u}{dx^2} + \left[\omega^2 - V(r)\right] u = 0, \qquad V(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right), \tag{1}$$

*Numerics reproducible from `rw_barrier.py`; the barrier reflectivity table ships as `rw_reflectivity.csv`.

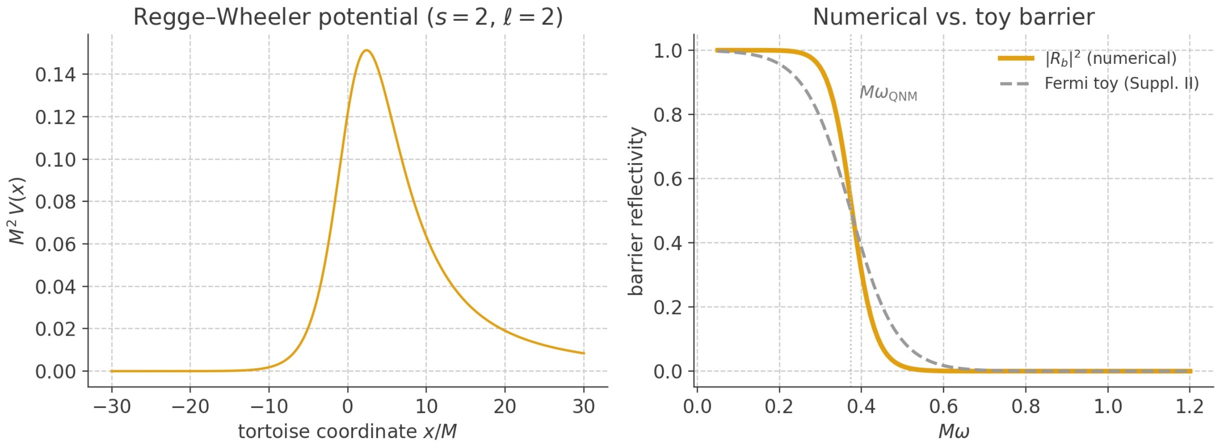


**Figure 1:** *Left:* the Regge–Wheeler potential on the tortoise axis. *Right:* numerical barrier reflectivity versus the Fermi toy of Supplement II. The toy was adequate near the quasinormal frequency but too soft in the wings.

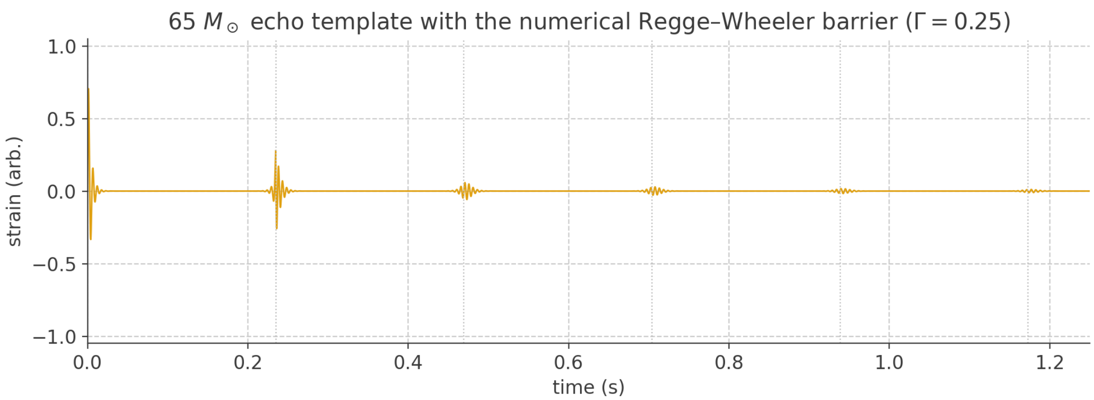


**Figure 2:** The 65 $M_\odot$ template with the exact barrier ($\Gamma = 0.25$). Compare Supplement II, Figure 3: morphology sharpened, delays and amplitudes essentially unchanged.

is solved numerically on the tortoise axis ($x = r + 2M \ln(r/2M - 1)$, inverted by Newton iteration), with a purely outgoing wave imposed at $x = +400M$ and the reflection/transmission coefficients of a wave incident from the horizon side — the configuration relevant for the echo cavity — extracted at $x = -80M$. Validation: the computed potential peak $M^2V_{\max} = 0.15129$ at $r = 3.281M$ matches the standard value ($0.1508$ at $r \simeq 3.28M$ [2]); scattering unitarity $|R_b|^2 + |T_b|^2 = 1$ holds to $6 \times 10^{-13}$ across the band; and the barrier is half-transparent at the fundamental quasinormal frequency, $|R_b|^2(M\omega = 0.374) = 0.531$, as expected since $M\omega_{\mathrm{QNM}}$ sits just below $\sqrt{M^2V_{\max}} = 0.389$.

Regenerating the 65 $M_\odot$ echo template with the numerical $R_b(\omega)$ changes the first-echo amplitude from 0.269 to 0.279 (at $\Gamma = 0.25$) and sharpens the echo pulse shapes (Figure 2); delays and comb spacing are untouched. Conclusion: the Supplement II template was robust, and the pipeline is now production-grade with two known omissions — spin (Teukolsky rather than Regge–Wheeler) and the physical value of $\Gamma$, to which we now turn.

## 3 The $\Gamma$ problem

Supplement II parameterized the wall by a coupling $\Gamma$ into the code subspace, $|R_w|^2 = 1 - \Gamma$, and treated $\Gamma = O(1)$ as natural. Deriving $\Gamma$ turns out to be the sharpest internal question the framework has produced, because the candidate answers differ by seventy-seven orders of magnitude in the exponent.

| **Scaling hypothesis** | **Physical assumption** | **Echo consequence** |
|---|---|---|
| (a) Dimension counting: $\Gamma \sim e^{S_{\text{mat}} - S_{BH}}$ | An exterior perturbation couples to $\text{ran}(A)$ with generic (Haar) overlap inside $\mathcal{H}_P$; $\dim \text{ran}(A) = e^{S_{\text{mat}}}$, $\dim \mathcal{H}_P = e^{S_{BH}}$ | $\Gamma \sim e^{-10^{77}}$ at $M_\odot$ ($S_{BH} = 1.05 \times 10^{77}$, $S_{\text{mat}} \sim 10^{58}$): no echoes, ever |
| (b) Funneling (**WHO-3**) | The boundary dynamics routes low-frequency exterior modes into the code subspace with $O(1)$ efficiency — a third, explicitly dynamical clause of the WHO condition | $\Gamma = O(1)$, frequency-flat, phase-coherent: the Supplement II template |
| (c) Thermalized boundary | The Planck region equilibrates at $T_H$ before re-emission; detailed balance | $\Gamma(\omega) = 1 - e^{-\hbar\omega/k_B T_H}$: the Boltzmann wall of [3] — Causal Symmetry becomes observationally degenerate with dissipative quantum horizons |

**Table 1:** Three candidate origins for the wall coupling. The framework does not currently select among them.

The honest summary: **observable flat-reflectivity echoes are not a generic prediction of Causal Symmetry.** Under genericity (row a) the wall is perfectly reflecting to exponential accuracy and the cavity is a mirror box — echoes exist but carry no imprint of the transfer channel; under thermalization (row c) the framework is degenerate with existing models. The distinctive Supplement II phenomenology stands or falls with funneling (row b), which we therefore promote to a named clause:

> **WHO-3 (funneling).** The boundary dynamics couples exterior perturbations into the code subspace with efficiency $\Gamma = O(1)$, frequency-flat at leading order.

This is a strong dynamical assumption — but it is now *isolated*: an echo detection with flat, phase-coherent reflectivity would be direct evidence for WHO-3; deep nulls constrain it; and a Boltzmann-shaped detection would favor thermalization and erase the framework's distinctive GW signature while leaving its information-theoretic core intact. A single free function $\Gamma(\omega)$ now carries the entire observational burden of the black-hole sector, which is exactly what a referee should be told.

## 4 The Page fork

Supplement I flagged the Page-curve tension as the framework's hardest problem: if island computations [4] are right that the radiation alone purifies, what does the Past Censor emit? The resolution is not a single answer but a structural dichotomy that maps exactly onto the lifetime fork of Supplement I (Figure 3).

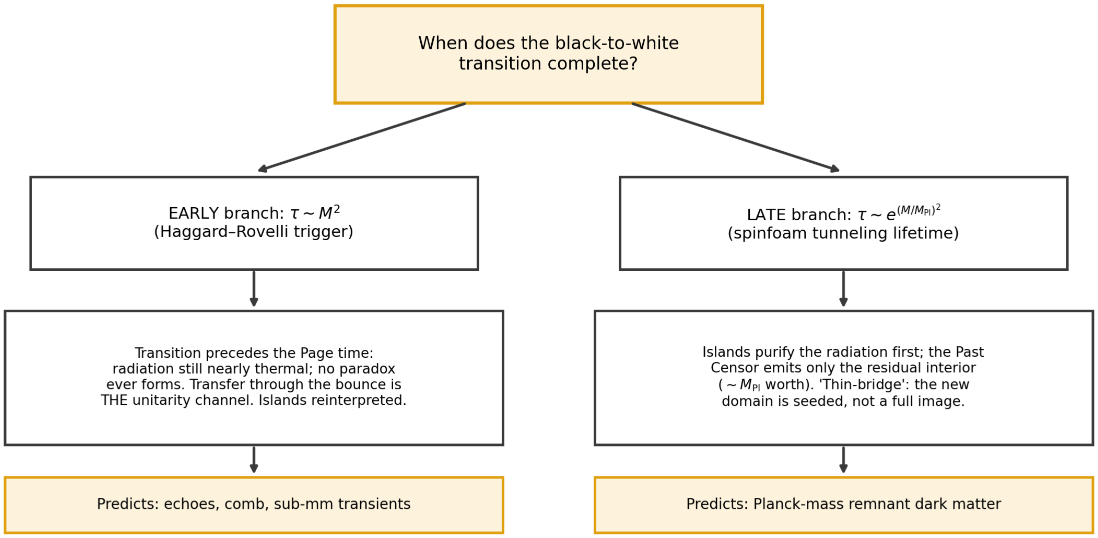


**Figure 3:** The Page fork. The framework's unitarity mechanism — and its observational program — depends on which lifetime branch nature selects. Both branches are internally consistent; they tell different stories and predict different phenomena.

**Early branch ($\tau \sim M^2$).** The transition completes long before the Page time ($\tau_{\rm Page} \sim \tau_H/2 \sim M^3$ vs. $\tau \sim M^2$). The paradox never forms: at transition the emitted radiation is genuinely thermal, and its purifier is not late radiation but the *emergent-domain partner state* — precisely the cross-domain thermofield double of Supplement I, Section 4, i.e. the parent paper's ER=EPR figure taken literally. On this branch, cross-domain transfer *replaces* the island mechanism for astrophysical black holes, and the framework must argue that island computations, which assume complete evaporation, simply do not describe bouncing holes. This is radical but coherent, and it is the branch on which the echo/comb/transient phenomenology lives.

**Late branch ($\tau \sim e^{(M/M_{\rm Pl})^2}$).** Islands operate as computed; the radiation purifies itself; by the endpoint the residual interior carries only $O(M_{\rm Pl})$-scale data. The Past Censor then emits a *thin bridge*: the emergent domain is seeded by Planck-scale residual information, not by a full image of the absorbed star. The low-entropy-origin mechanism survives untouched (a pure seed is a pure seed), but the parent papers' narrative — "information entering a black hole . . . reappears as initial conditions in a new domain" — must be weakened to "residual information seeds a new domain" on this branch. Any revision should either commit to a branch or present the fork as the framework's central open parameter.

We regard this reframing as the main conceptual yield of Supplement III: what looked like a fatal tension is actually the same fork already present in the lifetime scalings, now seen from the information side.

## 5 Energy bookkeeping, resolved by reframing

Within a single exterior (effective LQG collapse models), the bounce conserves the ADM mass: the white-hole phase has the same exterior mass function as the black-hole phase [5], so nothing is unaccounted. Across *causally disconnected* domains, by contrast, there is no shared asymptotic time and hence no covariant meaning to "the same energy": energy balance is not merely unverified but undefined. The frame-independent invariant that does cross the map $U$ is dimension — information content. The parent papers' instinct to write the global constraint as $d\mathcal{I}_{\rm tot}/dt = 0$ rather than as an energy statement is therefore correct as formulated, and we recommend making the point explicitly in any revision: *Causal Symmetry balances information, not energy, across domains; energy is conserved domain-locally in the usual way.* Objection 5 of Supplement I is hereby closed as "resolved by reframing."

## 6 The cosmological channel, estimated

Supplement II outlined the excited-initial-state program. The decisive number is the Bogoliubov occupation $|\beta_k|$ of emergent modes at the bounce. With the bounce scale set by the LQC critical density ($\Lambda \sim \rho_c^{1/4} \simeq 0.8\, M_{\rm Pl}$) and a Danielsson-type estimate $|\beta_k| \sim H/2\Lambda$ [6], inflation at $H \sim 10^{-5}$–$10^{-6}\, M_{\rm Pl}$ gives

$$2|\beta_k| \;\sim\; 10^{-6}\text{–}10^{-5} \tag{2}$$

for the fractional power-spectrum modulation — two to three orders below current sensitivity to features ($\sim 10^{-2}$–$10^{-3}$ [8]), with the folded-bispectrum enhancement of [7] improving matters but not closing the gap. Conclusion: the CMB channel is a consistency statement, not a near-term discriminator; the observational weight of the framework rests on the gravitational-wave and remnant sectors. The third loose end of Supplement I is hereby closed as "estimated, deprioritized."

## 7 Status after Supplement III

Of the framework's original open problems: the boundary map is formalized (I), quantified (II), and its barrier physics made exact (III); the WHO condition has three named clauses, each attackable; the lifetime fork and the Page fork are recognized as one fork seen from two sides; energy bookkeeping is closed; the CMB channel is bounded. What remains is genuinely hard: deriving $\Gamma(\omega)$ (equivalently, testing WHO-3) from spinfoam or other microphysics, and committing to a branch of the fork. These are field-level open problems, not gaps in this program's execution. The consolidated manuscript v2.0, prepared alongside this supplement, integrates Supplements I–III into a single referee-ready document with the speculative material quarantined.

## References


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