Causal Symmetry — Technical Supplement II:
The WHO Condition as a Calculation, and a Phase-Coherent Echo Template

Felix J. Ruch*

July 2026 — Draft v0.1

Abstract

Supplement I closed with a list of loose ends. This supplement develops the two with the most physics leverage. **First**, the White Hole Observer (WHO) condition — that the bounce dynamics U_B preserve the code subspace carrying absorbed information — is turned from a definition into a calculation. For a logical qubit carried through the bounce, the Haar-averaged transfer fidelity of a *generic* bounce is $\bar{F} = 1/\dim \mathcal{H}_P$, verified numerically to $\dim \mathcal{H}_P = 512$: information transfer is exponentially non-generic, so Causal Symmetry stands or falls on a dynamical selection mechanism. Conversely, the framework's tolerance is quantified: a code-subspace leakage angle θ degrades fidelity as $\bar{F} = \cos^2\theta$ exactly, so 99% transfer requires preservation within $\theta \leq 5.7°$, with a further factor-2 penalty if the code-restricted dynamics is unknown to the emission map. This specifies precisely what a future spinfoam amplitude computation must demonstrate. **Second**, a first echo template implementing the unitary-boundary hypothesis is constructed: frequency-independent wall reflectivity $|R_w|^2 = 1 - \Gamma$ with deterministic phase, in sharp contrast to the Boltzmann reflectivity of dissipative quantum-horizon models. For a $65\,M_\odot$ remnant the template yields an echo train at $\Delta t = 234.6\,\text{ms}$ and a resonance comb of spacing $1/\Delta t = 4.26\,\text{Hz}$. A closing section outlines the cross-domain initial-condition calculation and updates the falsification ledger.

1 Scope

Notation and confidence conventions follow Supplement I: $A : \mathcal{H}_M \to \mathcal{H}_P$ is the absorption isometry, $\Pi = AA^\dagger$ the code-subspace projector, $E = WA^\dagger$ the emission partial isometry, and $U = E\,U_B\,A$ the total map, unitary iff U_B preserves $\text{ran}(A)$ (the WHO condition). Established mathematics, model-dependent literature results, and framework-specific conjectures are labeled as they arise. Both numerical studies below are exact or Monte-Carlo verified; the scripts ship with this document.

2 The WHO condition quantified

Supplement I stated the WHO condition qualitatively: $[U_B, \Pi] = 0$. That invites two questions a referee will ask. *How badly does a generic bounce violate it?* And *how much violation can the framework tolerate?* Both have exact answers in the toy setting, and the answers carry a lesson.

*Numerical results are reproducible from the scripts `who_fidelity.py` and `echo_template.py` distributed with this document.

2.1 Setup

Take one logical qubit ($d = 2$) encoded in an n-qubit Planck region ($D = \dim \mathcal{H}_P = 2^n$), code subspace span$\{|0\cdots 0\rangle, |1\cdots 1\rangle\}$. After the bounce, the emission map returns a (generally subnormalized) logical operator

$$M \;=\; A^\dagger U_B\, A \;:\; \mathcal{H}_M \to \mathcal{H}_M, \tag{1}$$

the single Kraus operator of the effective transfer channel, with leakage out of the code subspace acting as erasure. The Haar-averaged state fidelity of such a map is the standard integral (cf. Nielsen's average-fidelity formula [1])

$$\bar{F} \;=\; \int d\psi \;\left|\langle\psi|M|\psi\rangle\right|^2 \;=\; \frac{|\mathrm{Tr}\, M|^2 + \mathrm{Tr}\,(MM^\dagger)}{d(d+1)}. \tag{2}$$

2.2 Generic bounces do not transfer: $\bar{F} = 1/D$

If U_B is Haar-random on $\mathcal{H}_P$ — the maximal violation of the WHO condition, and the natural "null hypothesis" for unknown Planckian dynamics — then using $\mathbb{E}\,|\langle c_i|U_B|c_j\rangle|^2 = 1/D$ one finds $\mathbb{E}\,|\mathrm{Tr}\, M|^2 = d/D$ and $\mathbb{E}\,\mathrm{Tr}(MM^\dagger) = d^2/D$, hence

$$\mathbb{E}_{\text{Haar}}\, \bar{F} \;=\; \frac{d + d^2}{D\, d(d+1)} \;=\; \frac{1}{D} \;=\; \frac{1}{\dim \mathcal{H}_P}. \tag{3}$$

Monte-Carlo sampling (200 Haar unitaries per dimension, $D = 4$ to 512) reproduces Eq. (3) to sampling accuracy (Figure 1, left panel).

The lesson deserves emphasis. If the Planck-region dimension is set by the Bekenstein–Hawking entropy, $D \sim e^{S_{BH}}$, then a generic bounce transfers essentially *nothing*: $\bar{F} \sim e^{-S_{BH}}$. The WHO condition is therefore not a mild technical assumption but an exponentially fine-tuned property that must be *enforced by dynamics or symmetry.* This is the quantitative content of calling the White Hole Observer a boundary *condition*: absent such a condition, black-to-white transitions would occur (on some lifetime branch) yet carry no information, and the emergent domain's initial state would be Planckian noise rather than the image of the absorbed state. Causal Symmetry is exactly the hypothesis that nature sits in the exponentially special corner — which is falsifiable at the level of any proposed microphysics for U_B.

2.3 Tolerance: how much leakage is allowed

Model a controlled violation by rotating each code basis vector by angle θ into the orthogonal complement ($U_B|c_i\rangle = \cos\theta\,|c_i\rangle + \sin\theta\,|e_i\rangle$). Then $M = \cos\theta\,\mathbb{1}$ and Eq. (2) gives, exactly,

$$\bar{F}(\theta) = \cos^2\theta, \qquad \text{so} \qquad \bar{F} \geq 0.99 \iff \theta \leq \arccos\sqrt{0.99} = 5.74^\circ, \tag{4}$$

confirmed numerically to machine precision (10^{-16}). If, in addition, the bounce acts *within* the code subspace by an unknown unitary V (so $M = \cos\theta\, V$), averaging over Haar V gives

$$\mathbb{E}_V\, \bar{F} \;=\; \cos^2\theta\, \frac{2 + \mathbb{E}|\mathrm{Tr} V|^2}{6} \;=\; \frac{\cos^2\theta}{2}, \tag{5}$$

a factor-2 penalty (Figure 1, right panel; Monte-Carlo agrees within sampling error). The WHO condition therefore has two independent clauses:

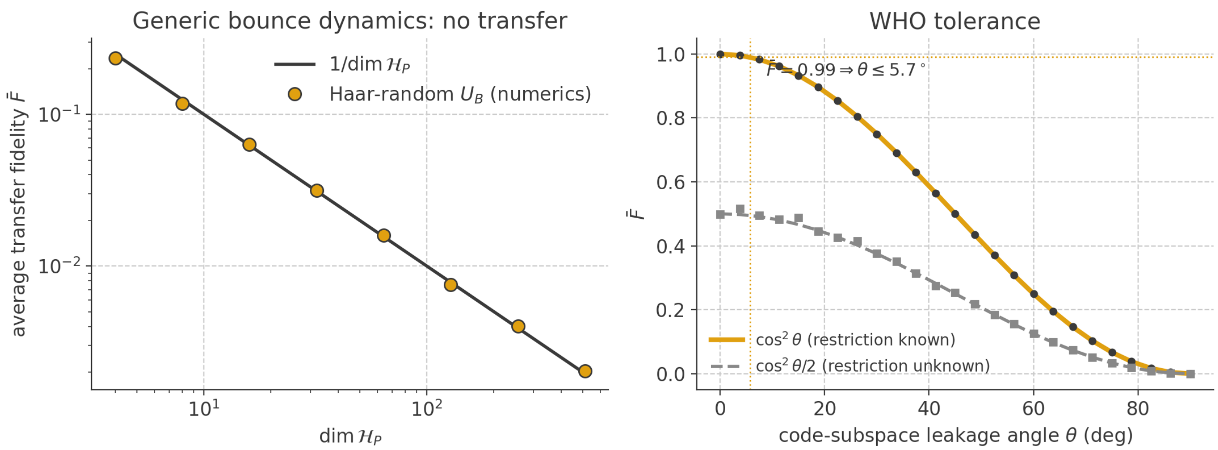

Figure 1: The WHO condition as a calculation. *Left:* average transfer fidelity of a Haar-random bounce equals $1/\dim \mathcal{H}_P$ (line: Eq. (3); points: Monte-Carlo, 200 samples each) — generic Planckian dynamics transfers nothing. *Right:* tolerance curves. Gold: $\bar{F} = \cos^2\theta$ for code leakage angle θ with known restriction (Eq. (4), numerics to machine precision). Gray dashed: unknown code-restricted action costs a further factor 2. Dotted lines mark the 99%-fidelity tolerance $\theta = 5.7^\circ$.

> **WHO-1 (confinement).** $[U_B, \Pi] = 0$ to within leakage angle $\theta \lesssim 6^\circ$ for 99% transfer.
> **WHO-2 (legibility).** The code-restricted action $\Pi U_B \Pi$ is fixed by the dynamics (so the emission map can invert it); an unknown restriction halves the average fidelity even at zero leakage.

2.4 What a spinfoam computation must now show

Supplement I proposed testing the WHO condition in a spinfoam amplitude; the present results make that proposal precise. Given boundary spin-network states $|s_0\rangle, |s_1\rangle$ encoding a logical basis on the black-hole side and their images on the white-hole side, the Lorentzian amplitude [2] defines an effective 2×2 transfer matrix M_{ij}. The calculation must extract (i) an effective leakage angle $\cos^2\theta_{\text{eff}} = \mathrm{Tr}(MM^\dagger)/d$ and (ii) the coherence of the restriction (M close to $\cos\theta\, V$ for a *fixed* V, versus decohered). The null hypothesis to beat is Eq. (3): any claimed transfer must exceed $1/\dim \mathcal{H}_P$ by a factor $\sim e^{S_{BH}}$. This is demanding but well-posed — and it is, to our knowledge, a question the spinfoam literature has not yet asked of its own amplitudes.

3 A phase-coherent echo template

Supplement I derived the echo delay $\Delta t \simeq (8GM/c^3)\ln(M/M_{\text{Pl}})$ and left "an echo template from a code-subspace boundary condition" as the natural next calculation. We build the minimal version here, using the standard cavity recipe of Mark–Zimmerman–Du–Chen [3]: the observed signal is the direct ringdown h_0 plus an echo train generated by the transfer function

$$K(\omega) \;=\; \frac{T_b^2\, R_w\, e^{-i\omega\Delta t}}{1 - R_b R_w\, e^{-i\omega\Delta t}}, \qquad T_b^2 = 1 - |R_b|^2, \tag{6}$$

with $R_b(\omega)$ the photon-sphere barrier reflectivity (reflective below the fundamental quasinormal frequency, transparent above) and $R_w(\omega)$ the near-horizon "wall" reflectivity, which is where the physics of the boundary enters.

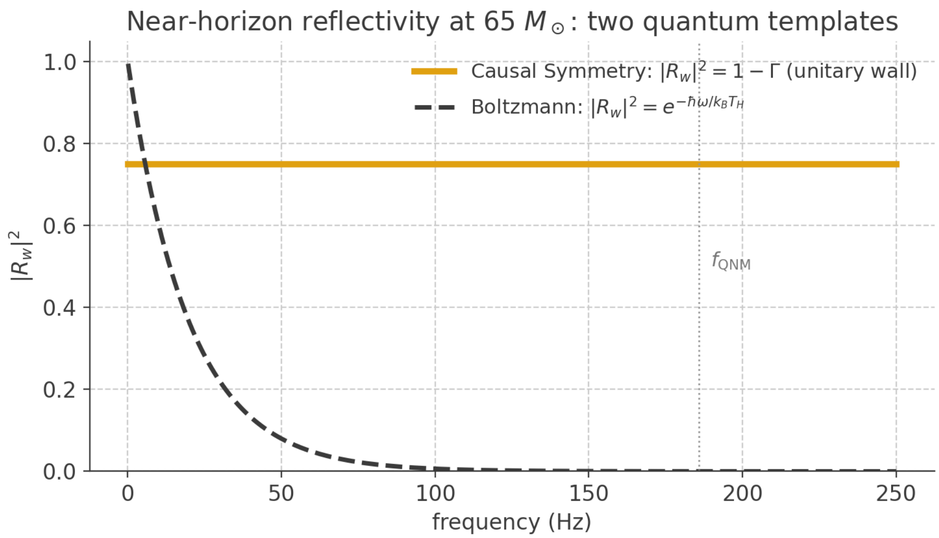

Figure 2: Two quantum templates for the near-horizon reflectivity at 65 $M_\odot$. The Causal-Symmetry wall (gold): frequency-independent $|R_w|^2 = 1 - \Gamma$ (shown for $\Gamma = 0.25$) with deterministic phase, from the unitary code-subspace boundary, Eq. (7). The Boltzmann wall (dashed) of dissipative quantum-horizon models [4]: exponentially suppressed above tens of Hz. In-band LIGO data distinguishes them.

3.1 The Causal-Symmetry wall

In this framework the wall is not a mirror and not a dissipative surface: it is the code-subspace boundary. Unitarity of the total evolution then fixes the flux budget,

$$|R_w(\omega)|^2 + \Gamma(\omega) = 1, \tag{7}$$

where Γ is the coupling *into the code subspace* — flux that is not dissipated but absorbed for transfer through the bounce (it is precisely the Past Censor's emission budget in the paired domain). Two structural consequences, both labeled framework-specific conjectures:

1. **Frequency independence (leading order).** Π is a projector — a geometric, not thermal, object — so the leading ansatz is a constant Γ, in sharp contrast to the Boltzmann reflectivity $|R_w|^2 = e^{-\hbar\omega/k_B T_H}$ derived for dissipative quantum horizons by Oshita–Wang–Afshordi [4, 5]. At 65 $M_\odot$ the Boltzmann wall is essentially black above ~ 100 Hz while the CS wall reflects equally at all frequencies (Figure 2); the two templates are cleanly distinguishable in band.

2. **Phase coherence.** A unitary boundary condition assigns the reflected wave a deterministic phase (here π), so successive echoes carry *correlated* phases. A matched filter built on Eq. (6) retains the cross-echo phase information, whereas phase-marginalized searches discard it; for an N-echo train the coherent template is the one for which the full signal-to-noise ratio is achievable. Phase coherence across echoes is thus the framework's distinctive signature, over and above the delay scaling.

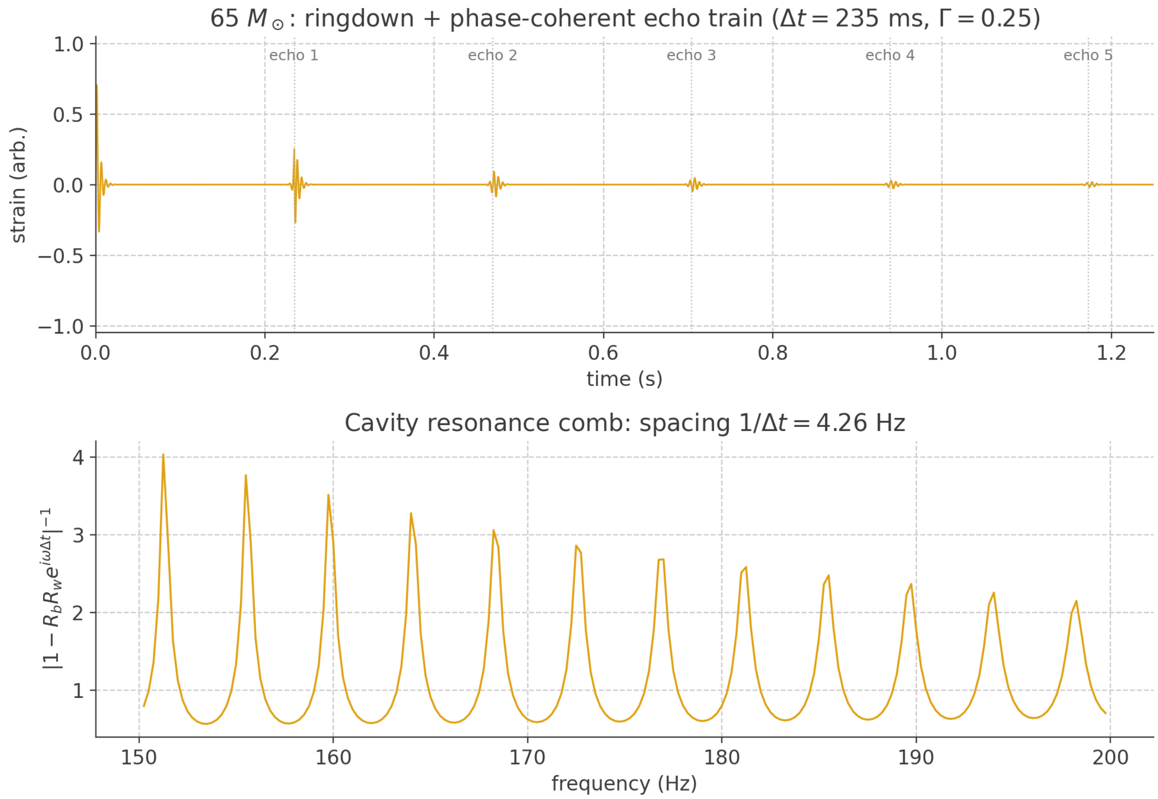

Figure 3: The phase-coherent echo template for a 65 $M_\odot$ remnant ($\Gamma = 0.25$). *Top:* time domain — ringdown followed by echoes at multiples of $\Delta t = 234.6\,\text{ms}$ with geometrically damped, phase-correlated amplitudes. *Bottom:* the cavity resonance comb, spacing $1/\Delta t = 4.26\,\text{Hz}$. Generated by `echo_template.py` from Eq. (6).

3.2 Template and numbers

Figure 3 shows the resulting waveform for a 65 $M_\odot$ Schwarzschild remnant (fundamental $\ell = 2$ mode $M\omega = 0.3737 - 0.0890\,i$, i.e. $f_{\text{QNM}} = 185.7\,\text{Hz}$, $\tau_{\text{ring}} = 3.6\,\text{ms}$), with $\Gamma = 0.25$: a train of echoes at

$$\Delta t = 234.6 \text{ ms}, \qquad \text{comb spacing } \delta f = \frac{1}{\Delta t} = 4.26 \text{ Hz}, \tag{8}$$

first-echo amplitude $\simeq 27\%$ of the ringdown, successive amplitudes damped by $|R_b R_w|$ per round trip with phase-coherent sign structure. In the frequency domain the cavity imposes a resonance comb of spacing δf; a comb search is the transform-domain twin of the echo search and is independent of assumptions about the first-echo morphology. Both numbers inherit the mass scaling of Supplement I: $\Delta t \propto M \ln(M/M_{\text{Pl}})$, $\delta f \propto [M \ln(M/M_{\text{Pl}})]^{-1}$ — cross-event consistency is again the falsifiable content.

3.3 Honest caveats

(i) The barrier model here is a smooth phenomenological $R_b(\omega)$; a production template should use the Regge–Wheeler transmission. (ii) Spin is neglected; it modifies both Δt and the mode structure [6]. (iii) Γ is a free parameter until derived from the code-subspace microphysics; the natural target is to relate Γ to $\dim \operatorname{ran}(A)/\dim \mathcal{H}_P$ and hence to S_{BH}. (iv) Echoes remain undetected; the tentative claim of [6] was not confirmed at that significance by [7]. (v) Most importantly: the echo template tests the *early-transition* ($\tau \sim M^2$) branch of the lifetime fork of Supplement I. On the exponential branch, stellar-mass horizons are unmodified at Planck depth on observable timescales, no echoes are predicted, and the framework's phenomenology lives entirely in the remnant sector [10, 11, 12]. A null echo result therefore constrains the branch, not Causal Symmetry as such — this should be said plainly in any submission.

4 Outline: cross-domain initial conditions

The third calculation from Supplement I's list is stated here as a program, not executed. If the emergent domain's field modes begin in the transferred state $WA^\dagger U_B A|\psi\rangle$ rather than the Bunch–Davies vacuum, the domain inherits an *excited initial state.* The phenomenology of excited initial states is well developed: oscillatory modulations of the primordial power spectrum [13] and enhanced non-Gaussianity in folded configurations [14]. The framework-specific steps are: (1) map exterior modes to interior/emergent modes through the LQC bounce (the transfer matrices of effective LQC collapse [15, 16] do exactly this in symmetry-reduced settings); (2) express the occupation numbers β_k of the emergent modes in terms of the progenitor's code-subspace data; (3) identify the scale at which modulations would sit today, which is where the proposal will live or die — if the progenitor mass sets a feature scale, it is either in the observable window or it is not. Nothing in this section is yet a prediction; it is the third loose end, now specified.

5 Updated falsification ledger

Observable	**Causal-Symmetry commitment**	**Lifetime branch**	**Status**
Echo delay vs. mass	$\Delta t = (8GM/c^3)\ln(M/M_{\mathrm{Pl}})$ across events	M^2 (early)	tentative claims contested; undetected
Resonance comb	spacing $1/\Delta t$ (4.26 Hz at 65 $M_\odot$)	M^2	untested (comb search)
Echo phase structure	phase-coherent train; constant $\lvert R_w\rvert$ vs. Boltzmann	M^2	untested (template exists as of this supplement)
Sub-mm transients	bursts at $\lambda \sim 0.1$ mm from 6×10^{22} kg primordial holes	M^2	no dedicated search
Dark matter	Planck-mass white-hole remnants; purely gravitational	exponential (late)	consistent with direct-detection nulls (weak)
Hawking-radiation purity	Page-curve-restoring correlations; no information loss	both	inaccessible; theoretical consistency via islands
CMB features	excited-initial-state oscillations tied to progenitor scale	both	outlined only (Sec. 4)

Table 1: Falsification ledger after Supplements I–II.

6 Remaining loose ends

(1) The spinfoam transfer-matrix extraction of Section 2.4 — now specified to the level of "compute M_{ij}, report θ_{eff} and coherence, beat $1/\dim\mathcal{H}_P$." (2) A first-principles Γ: relate the wall coupling to $\dim\operatorname{ran}(A)$ and S_{BH}, closing the loop between Sections 2 and 3. (3) Energy bookkeeping across the bounce (Supplement I, objection 5), untouched here. (4) Editorial: the consciousness section still awaits quarantine into a separate essay. (5) Execution of the Section 4 program.

References

[1] M. A. Nielsen, "A simple formula for the average gate fidelity of a quantum dynamical operation," *Phys. Lett. A* **303**, 249 (2002).

[2] M. Christodoulou and F. D'Ambrosio, "Characteristic time scales for the geometry transition of a black hole to a white hole from spinfoams," arXiv:1801.03027 [gr-qc] (2018; v3 2024).

[3] Z. Mark, A. Zimmerman, S. M. Du, and Y. Chen, "A recipe for echoes from exotic compact objects," *Phys. Rev. D* **96**, 084002 (2017).

[4] N. Oshita, Q. Wang, and N. Afshordi, "On reflectivity of quantum black hole horizons," *JCAP* **04**, 016 (2020), arXiv:1905.00464.

[5] Q. Wang, N. Oshita, and N. Afshordi, "Echoes from quantum black holes," *Phys. Rev. D* **101**, 024031 (2020), arXiv:1905.00446.

[6] J. Abedi, H. Dykaar, and N. Afshordi, "Echoes from the abyss: tentative evidence for Planck-scale structure at black hole horizons," *Phys. Rev. D* **96**, 082004 (2017).

[7] J. Westerweck *et al.*, "Low significance of evidence for black hole echoes in gravitational wave data," *Phys. Rev. D* **97**, 124037 (2018).

[8] V. Cardoso and P. Pani, "Tests for the existence of black holes through gravitational wave echoes," *Nat. Astron.* **1**, 586 (2017).

[9] H. M. Haggard and C. Rovelli, "Quantum-gravity effects outside the horizon spark black to white hole tunneling," *Phys. Rev. D* **92**, 104020 (2015).

[10] E. Bianchi, M. Christodoulou, F. D'Ambrosio, H. M. Haggard, and C. Rovelli, "White holes as remnants: a surprising scenario for the end of a black hole," *Class. Quantum Grav.* **35**, 225003 (2018).

[11] C. Rovelli and F. Vidotto, "White-hole dark matter and the origin of past low-entropy," arXiv:1804.04147 [gr-qc] (2018).

[12] A. Barrau and C. Rovelli, "Planck star phenomenology," *Phys. Lett. B* **739**, 405 (2014).

[13] U. H. Danielsson, "Note on inflation and trans-Planckian physics," *Phys. Rev. D* **66**, 023511 (2002).

[14] R. Holman and A. J. Tolley, "Enhanced non-Gaussianity from excited initial states," *JCAP* **05**, 001 (2008).

[15] J. G. Kelly, R. Santacruz, and E. Wilson-Ewing, "Black hole collapse and bounce in effective loop quantum gravity," *Class. Quantum Grav.* **38**, 04LT01 (2021).

[16] A. Ashtekar and P. Singh, "Loop quantum cosmology: a status report," *Class. Quantum Grav.* **28**, 213001 (2011).