A Quantum-Causal Model of Existence and Time

The Hypothesis of Causal Symmetry with a Quantified Boundary Map
Version 2.0 (consolidated)

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Abstract

We develop the *Principle of Causal Symmetry*: gravitational absorption at black-hole boundaries ("Future Censors") is balanced by emission at white-hole boundaries ("Past Censors") in causally disconnected domains, with a unitary map across a quantum bounce conserving global information while each domain retains a local thermodynamic arrow. Version 2.0 replaces the definitional equations of the original papers with a worked formal structure: absorption is an isometry A, emission is the partial isometry $E = WA^\dagger$ (the inverse of A on its image), and global unitarity is equivalent to a three-clause boundary condition (the White Hole Observer, WHO) on the bounce dynamics — confinement to the code subspace, legibility of the code-restricted action, and funneling of exterior perturbations. The condition is quantified: a Haar-random bounce transfers information with fidelity $1/\dim \mathcal{H}_P \sim e^{-S_{BH}}$, so Causal Symmetry is the hypothesis of an exponentially non-generic, dynamically enforced corner; 99% transfer tolerates code-subspace leakage below 5.7°. Phenomenology is developed on both branches of the black-to-white lifetime fork: on the early ($\tau \sim M^2$) branch, gravitational-wave echoes with delay $\Delta t \simeq (8GM/c^3)\ln(M/M_{\rm Pl})$, a resonance comb of spacing $1/\Delta t$, phase-coherent reflectivity distinguishable from Boltzmann templates, and sub-millimeter transients from $\sim 6 \times 10^{22}$ kg primordial holes; on the late (exponential) branch, Planck-mass white-hole remnant dark matter. The echo amplitude is controlled by a single function $\Gamma(\omega)$ whose three candidate scalings differ by seventy-seven orders of magnitude in the exponent; observable echoes are a measurement of the funneling clause, not a generic prediction. The Page-curve tension resolves into the same fork: cross-domain transfer either replaces the island mechanism (early) or emits a Planck-scale residual seed (late). We compare with conformal cyclic cosmology — which requires the information loss this framework forbids — and with the two-arrow cosmologies that are its nearest relatives, and we state plainly what would falsify the program.

1 Introduction

Three puzzles motivate this work: the black-hole information paradox [3, 4], the local arrow of time despite time-reversible microphysics, and the low-entropy beginning. The Causal Symmetry hypothesis addresses all three with one structural idea: absorption in one causal domain is balanced by emission in another, so that the global information budget is exactly conserved,

$$\frac{d\mathcal{I}_{\rm tot}}{dt} = 0, \tag{1}$$

*This version consolidates the October 2025 papers with Technical Supplements I–III (July 2026). Numerical results are reproducible from the scripts listed in Appendix A. The speculative material of the original Section 9 has been removed to a separate essay.

while each domain separately obeys the second law, $\Delta S \geq 0$. Black holes act as *Future Censors* (absorption boundaries); white holes act as *Past Censors* (time-reversed emission boundaries); a quantum bounce of the loop-quantum-cosmology type [5] provides the nonsingular bridge; and a non-local boundary condition — the *White Hole Observer* (WHO) — enforces the mapping.

The original papers (October 2025) stated this at the level of principle. Version 2.0 consolidates three technical supplements into a single document and is different in kind: every equation below is either derived, numerically verified, or explicitly labeled a conjecture. The main additions are: the refinement of the emission channel from the ill-defined $E = A^{-1}$ to a partial isometry (Section 2); worked toy models separating conservation from local irreversibility (Section 3); a quantification of how special the WHO condition is (Section 4); the black-to-white lifetime fork and its phenomenology on both branches (Sections 5–7); the Γ problem, which locates the entire gravitational-wave burden of the framework in one derivable function (Section 6); the Page fork (Section 8); and a sharpened comparison with rival cyclic and two-arrow frameworks (Section 9). Speculative material on observers and classicality has been removed to a separate essay.

Throughout, three confidence levels are distinguished: **established** results independent of this framework; **model-dependent** literature results; and **conjectures** specific to Causal Symmetry.

2 The principle and the boundary map

2.1 From $E = A^{-1}$ to a partial isometry

A quantum channel admits a channel inverse if and only if it is unitary, so the original papers' emission relation $E = A^{-1}$ cannot hold literally for absorption into a larger boundary space. The consistent structure uses isometries. Absorption into the Planck-region Hilbert space $\mathcal{H}_P$ is

$$A : \mathcal{H}_M \to \mathcal{H}_P, \qquad A^\dagger A = \mathbb{1}_M, \qquad \Pi \equiv AA^\dagger, \tag{2}$$

with Π the projector onto the *code subspace* $\mathrm{ran}(A)$ — the language of holographic quantum error correction [14], which we adopt deliberately. Emission is

$$E \equiv WA^\dagger : \mathcal{H}_P \to \mathcal{H}_{M'}, \qquad EA = W, \qquad E^\dagger E = \Pi, \tag{3}$$

where W (possibly antiunitary, absorbing time reversal) identifies mode bases across domains: E inverts A *on its image* and annihilates the complement. With bounce dynamics $U_B : \mathcal{H}_P \to \mathcal{H}_P$, the total map is

$$U \;\equiv\; E\,U_B\,A \;=\; W\,(A^\dagger U_B A) :\; \mathcal{H}_M \to \mathcal{H}_{M'}. \tag{4}$$

2.2 The WHO condition, in three clauses

U is unitary — Eq. (1) holds exactly — precisely when the bounce dynamics respects the code subspace. Developing the framework has resolved this requirement into three separately attackable clauses:

> **WHO-1 (confinement).** $[U_B, \Pi] = 0$: absorbed information stays in $\mathrm{ran}(A)$ through the bounce.
>
> **WHO-2 (legibility).** The code-restricted action $\Pi U_B \Pi$ is fixed by the dynamics, so the emission map can invert it.
>
> **WHO-3 (funneling).** The boundary couples exterior perturbations into the code subspace with $O(1)$ efficiency (this clause carries the gravitational-wave phenomenology; see Section 6).

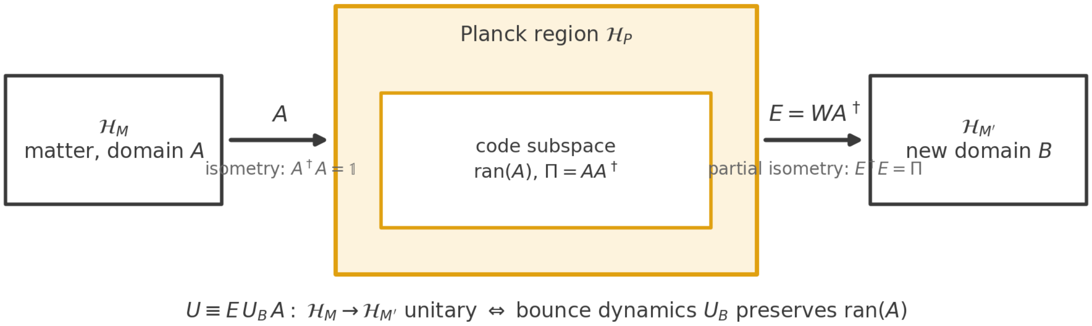

Figure 1: The boundary map. Absorption A is an isometry into the Planck region; emission $E = WA^\dagger$ is a partial isometry; the total map $U = E\,U_B\,A$ is unitary iff the bounce dynamics preserves the code subspace.

The White Hole Observer is thus not an agent but a boundary condition: the statement that nature implements WHO-1–3. Sections 4 and 6 quantify how strong a statement that is.

3 Toy realizations

Two minimal models factorize the framework's claims.

3.1 Conservation: a qubit code subspace

Let $\mathcal{H}_M = \mathbb{C}^2$, $\mathcal{H}_P = (\mathbb{C}^2)^{\otimes 3}$, $A|0\rangle = |000\rangle$, $A|1\rangle = |111\rangle$, $W = \mathbb{1}$. For any U_B obeying WHO-1–2 (e.g. a relative phase within the code), the infalling state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ re-emerges in the new domain as $\alpha|0'\rangle + e^{i\theta}\beta|1'\rangle$: unit-fidelity transfer, global purity throughout, Eq. (1) exact. An observer confined to the collapse domain, tracing out the transferred degrees of freedom, sees a mixed state: information is locally inaccessible, globally intact.

3.2 Local irreversibility: the cross-domain thermofield double

For a field mode of frequency ω, the two-mode squeezed state

$$|\Phi\rangle = \sqrt{1-\lambda^2}\sum_n \lambda^n |n\rangle_A |n\rangle_B, \qquad \lambda = e^{-\omega/2T_H}, \qquad T_H = \frac{1}{8\pi G M}, \tag{5}$$

makes each side exactly thermal at the Hawking temperature ($S_A > 0$, the local second law) while the joint state is pure ($S_{AB} = 0$). The emergent domain's start has zero fine-grained entropy *because* it is the purification of what the other domain absorbed — the framework's answer to the past hypothesis, echoing the perspectival reading of past low entropy in [12]. Eq. (5) is also the state to which Maldacena–Susskind assign a wormhole dual [13]: the ER=EPR bridge between domains is not a cartoon but this state's standard geometric reading.

4 How special is the boundary condition?

The WHO condition can be quantified, and the answer carries the framework's central lesson. For a logical qubit encoded in $\mathcal{H}_P$ with $\dim \mathcal{H}_P = D$, the effective transfer operator is $M = A^\dagger U_B A$ and

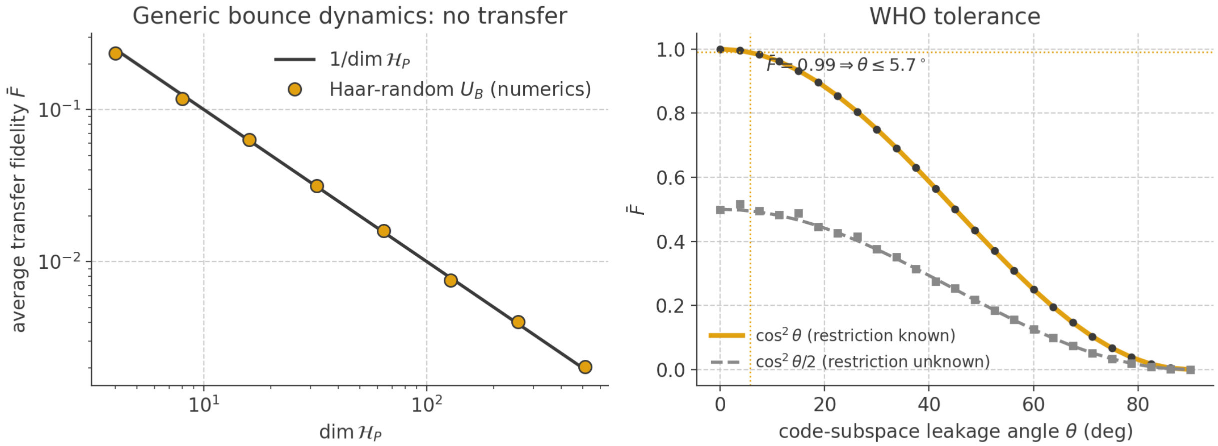

Figure 2: The WHO condition quantified. *Left:* Haar-random bounce dynamics transfers with fidelity $1/\dim\mathcal{H}_P$ (line: Eq. (6); points: Monte-Carlo). *Right:* tolerance curves for leakage angle θ, with the factor-2 penalty for an unknown code-restricted action.

its Haar-averaged state fidelity is $\bar{F} = \left(|\mathrm{Tr}M|^2 + \mathrm{Tr}MM^\dagger\right)/6$ [17]. If U_B is Haar-random — the null hypothesis for unknown Planckian dynamics — then

$$\mathbb{E}_{\mathrm{Haar}}\,\bar{F} = \frac{1}{D} \;\sim\; e^{-S_{BH}}, \tag{6}$$

verified by Monte-Carlo to $D = 512$ (Figure 2, left). A generic bounce transfers *nothing*: black-to-white transitions would occur, yet seed the new domain with Planckian noise. Causal Symmetry is exactly the hypothesis that dynamics or symmetry enforces the exponentially special corner — a statement falsifiable against any proposed microphysics for U_B. Conversely the tolerance is known exactly: code-subspace leakage by angle θ gives $\bar{F} = \cos^2\theta$ (so 99% transfer requires $\theta \leq 5.7^\circ$), and an unknown code-restricted action halves the average fidelity even at zero leakage (Figure 2, right) — the quantitative content of WHO-1 and WHO-2 respectively.

For spinfoam microphysics this defines a concrete target: from Lorentzian transition amplitudes [7] between boundary states encoding a logical basis, extract the effective 2×2 transfer matrix, report the effective leakage angle and the coherence of the restriction, and beat the decoupling baseline of Eq. (6) by $\sim e^{S_{BH}}$. To our knowledge this question has not been asked of existing amplitudes.

5 Bounce microphysics and the lifetime fork

Two quantum-gravity inputs anchor the bounce. Effective loop quantum cosmology gives the nonsingular kinematics,

$$H^2 = \frac{8\pi G}{3}\,\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \rho_c \approx 0.41\,\rho_{\mathrm{Pl}}, \tag{7}$$

robust within its symmetry class [5] and extended to collapse spacetimes by effective models [8]. Covariant spinfoam amplitudes supply candidate microphysics and rates for the black-to-white transition [6, 7]. The two are complementary where this framework needs them: LQC guarantees that "through the bounce" is a well-defined place for U to live; spinfoams are the only current source of numbers for U_B.

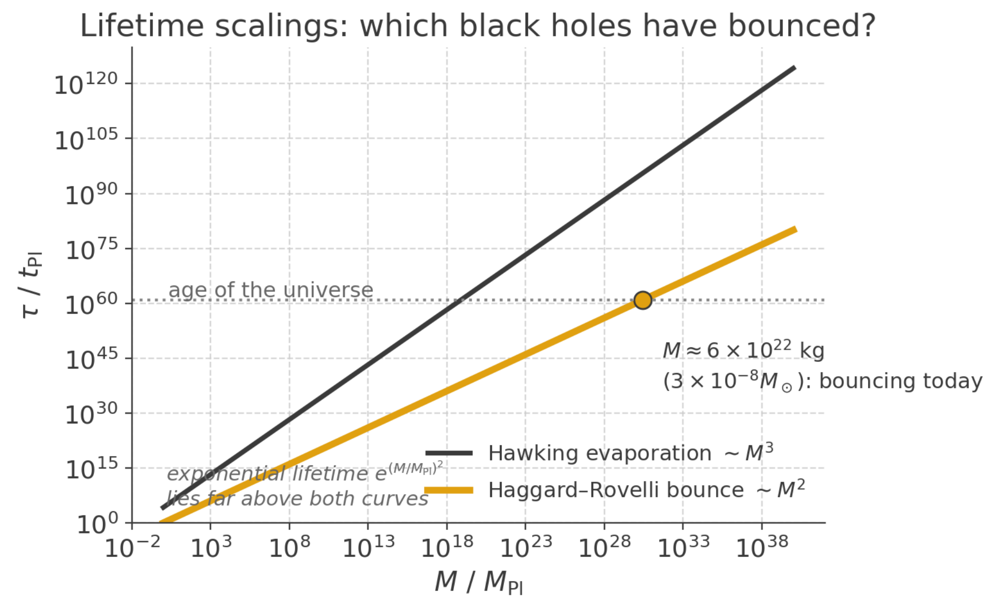

Figure 3: Lifetime scalings in Planck units. Under $\tau \sim M^2$ (gold), holes of $\approx 6 \times 10^{22}$ kg bounce today; under Hawking evaporation $\sim M^3$ (dark), only $\sim 10^{12}$ kg holes end today; the exponential spinfoam lifetime lies far above both for macroscopic masses.

The transition's *lifetime* is contested, and the contest matters observationally. The Haggard–Rovelli trigger argument gives $\tau \sim (M/M_{\rm Pl})^2 t_{\rm Pl}$ [6]; the revised spinfoam estimate gives a crossing time linear in M but a lifetime exponential, $\tau \sim e^{(M/M_{\rm Pl})^2} t_{\rm Pl}$ [7]. The fork is not a nuisance but the framework's central observational parameter (Figure 3):

- **Early branch ($\tau \sim M^2$):** primordial holes of $M_* \approx 6 \times 10^{22}$ kg ($3 \times 10^{-8} M_\odot$) complete their bounce at the present cosmic age, emitting at wavelengths of order their horizon size, $\lambda \sim 0.1$ mm [10]; stellar-mass horizons carry Planck-depth structure and the echo phenomenology of Section 6.
- **Late branch (exponential):** no astrophysical hole has transitioned; the framework's phenomenology lives in Planck-mass white-hole remnants [11], proposed as a purely gravitational dark-matter component [12].

6 Gravitational-wave phenomenology

6.1 Echo delay and resonance comb

If the bounce replaces the horizon with structure at proper Planck depth, ingoing perturbations reflect between it and the photon-sphere barrier. The tortoise-coordinate round trip gives, to leading logarithmic accuracy,

$$\Delta t_{\rm echo} \simeq \frac{8GM}{c^3} \ln \frac{M}{M_{\rm Pl}} \qquad \Longrightarrow \qquad \delta f = \frac{1}{\Delta t} \ \ \text{(resonance comb)}, \tag{8}$$

the $8M \ln M$ delay of [18]. Numerically: 35, 107, 235, 363 ms at 10, 30, 65, 100 $M_\odot$ (Figure 4), with comb spacing 4.26 Hz at 65 $M_\odot$. The scaling — linear in M with a slowly varying logarithm $\simeq 90$ —

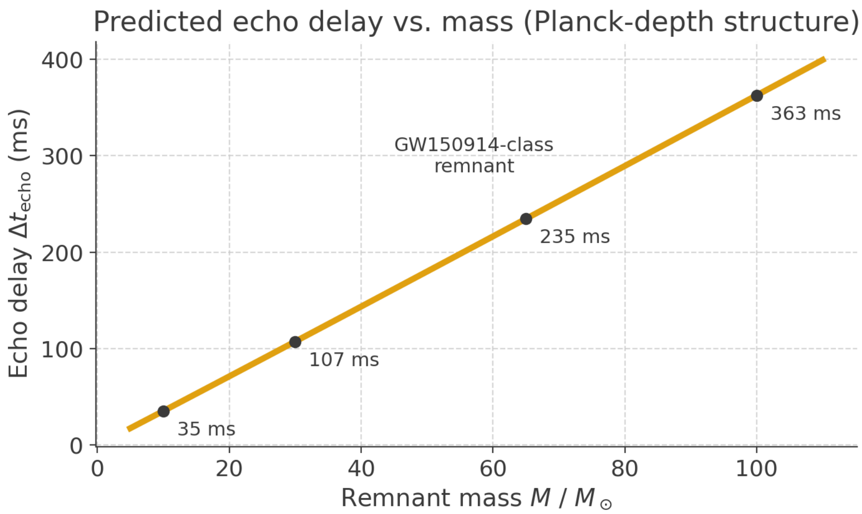

Figure 4: Echo delay versus remnant mass, Eq. (8).

is the cross-event prediction; current observational status is honestly summarized as tentative claims [18] not confirmed at that significance by independent reanalysis [19], with the phenomenology reviewed in [20, 21].

6.2 A production-grade template

The template is built from the standard cavity transfer function [21] with the photon-sphere barrier computed by numerically solving the Regge–Wheeler equation ($s = 2$, $\ell = 2$; potential peak validated against [25] to 0.3%, scattering unitarity to 10^{-13}; the barrier is half-transparent at the fundamental quasinormal frequency, $|R_b|^2 = 0.53$). The wall is the framework-specific element: unitarity of the boundary fixes $|R_w(\omega)|^2 + \Gamma(\omega) = 1$ with Γ the coupling *into the code subspace* — flux absorbed for transfer, not dissipated. Two labeled conjectures follow: frequency-flat Γ at leading order (a projector is geometric, not thermal), and deterministic reflection phase, hence a *phase-coherent* echo train — cleanly distinguishable in band from the Boltzmann reflectivity $|R_w|^2 = e^{-\hbar\omega/k_B T_H}$ of dissipative quantum horizons [22, 23], which is essentially black above $\sim 100\,\mathrm{Hz}$ at $65\,M_\odot$ (Figure 5). The resulting waveform (Figure 6) has first-echo amplitude 0.28 of the ringdown at $\Gamma = 0.25$.

6.3 The Γ problem

Deriving Γ is the sharpest internal question the framework has produced, because the candidate scalings differ by seventy-seven orders of magnitude in the exponent (Table 1). Under generic (Haar) overlap, dimension counting gives $\Gamma \sim e^{S_{\rm mat} - S_{BH}} \approx e^{-10^{77}}$ at a solar mass ($S_{BH} = 1.05 \times 10^{77}$, $S_{\rm mat} \sim 10^{58}$): the wall is a mirror to exponential accuracy and echoes carry no imprint of the transfer channel. An $O(1)$, frequency-flat Γ is exactly the funneling clause WHO-3. A thermalized boundary reduces to the Boltzmann wall. **Observable flat-reflectivity echoes are therefore not a generic prediction of Causal Symmetry but a measurement of WHO-3** — and deep nulls constrain that clause specifically, leaving the information-theoretic core intact. This honesty defines the stakes of the template: one detection with flat, phase-coherent reflectivity would be direct evidence for dynamical funneling into a code subspace.

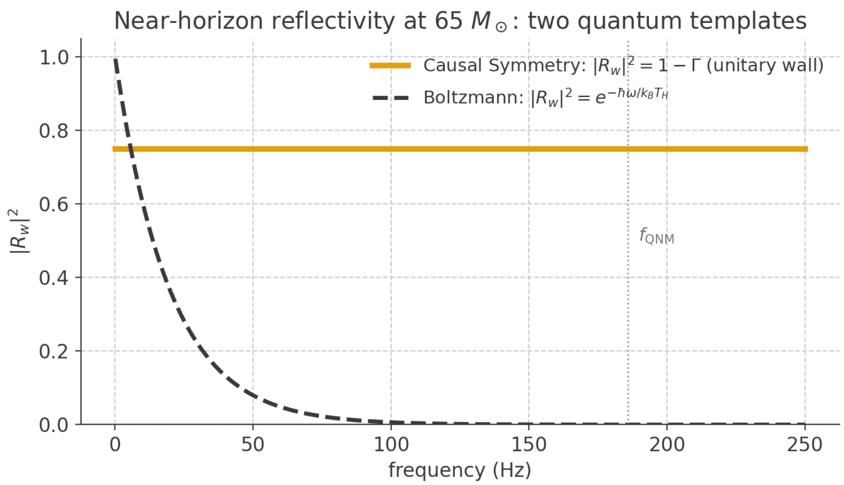

Figure 5: Two quantum templates for the wall at 65 $M_\odot$: frequency-flat Causal-Symmetry reflectivity (gold, $\Gamma = 0.25$) versus Boltzmann (dashed).

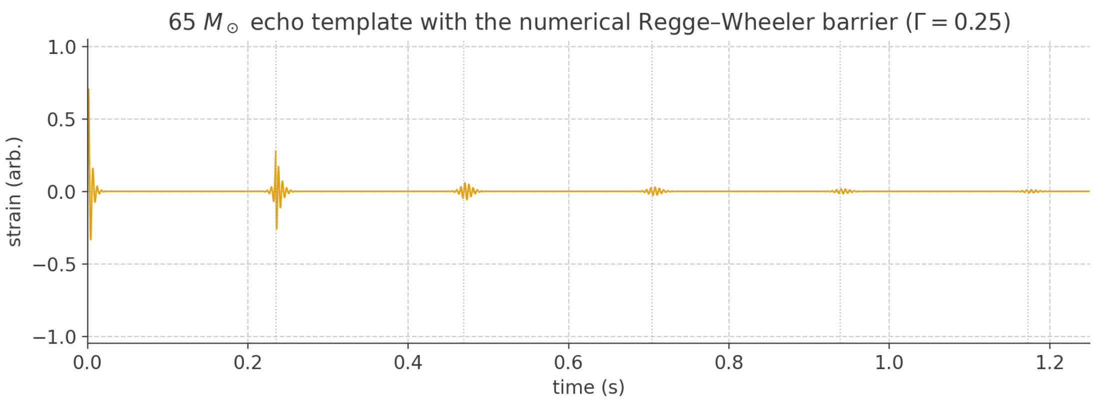

Figure 6: The 65 $M_\odot$ echo template with the numerical Regge–Wheeler barrier ($\Gamma = 0.25$): echoes at multiples of 234.6 ms, phase-coherent, geometrically damped.

7 Other channels

Sub-millimeter transients (early branch). Holes of 6×10^{22} kg bouncing today emit at $\lambda \sim r_s \sim 0.1$ mm (∼THz), with a possible higher-energy component set by the collapse temperature [10]; no dedicated transient search exists in this band.

Remnant dark matter (late branch). Planck-mass white-hole remnants [11, 12] interact only gravitationally — consistent with, though not evidenced by, direct-detection nulls.

Cosmological features. If the emergent domain's modes exit the bounce excited, excited-initial-state phenomenology applies: power-spectrum oscillations [26] and folded-limit non-Gaussianity [27]. A Danielsson-type estimate with the cutoff at the LQC bounce scale gives modulations $2|\beta_k| \sim 10^{-6}$–10^{-5} for $H \sim 10^{-6}$–$10^{-5} M_{\rm Pl}$ — two to three orders below current feature sensitivity [28]. The CMB channel is a consistency statement, not a near-term discriminator.

Hawking-radiation purity. On either branch the framework forbids information loss; eventual purity-restoring correlations in late radiation [4] are the in-principle (though practically inaccessible)

Scaling	Assumption	Echo consequence
$\Gamma \sim e^{S_{\rm mat} - S_{BH}}$	generic (Haar) overlap with the code subspace	$e^{-10^{77}}$ at $M_\odot$: perfect mirror; no transfer imprint
$\Gamma = O(1)$, flat	**WHO-3** funneling (dynamical)	the phase-coherent template of Fig. 6
$\Gamma = 1 - e^{-\hbar\omega/k_B T_H}$	thermalized boundary	degenerate with Boltzmann quantum horizons [22]

Table 1: The Γ problem: three candidate origins of the wall coupling.

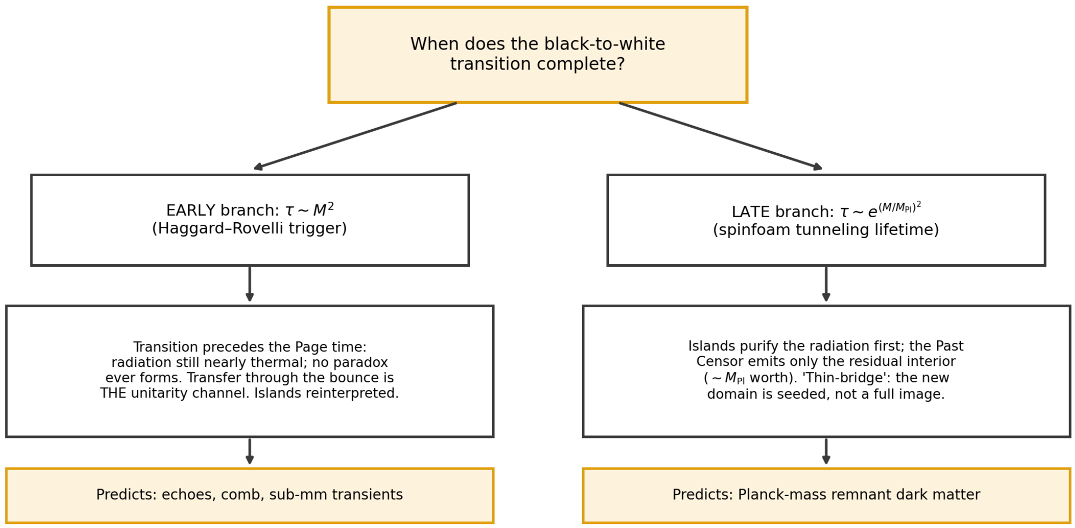

Figure 7: The Page fork: the framework's unitarity mechanism and observational program on the two lifetime branches.

commitment.

8 The Page fork

If island computations [16] are right that radiation alone purifies, what does the Past Censor emit? The answer is the lifetime fork seen from the information side (Figure 7). On the **early branch** the transition precedes the Page time; the paradox never forms; the radiation's purifier is the emergent-domain partner (the cross-domain thermofield double of Eq. (5)), and cross-domain transfer *replaces* the island mechanism for bouncing holes — radical, but coherent, since island results assume complete evaporation. On the **late branch** islands operate as computed and the Past Censor emits a *thin bridge*: a Planck-scale residual seed rather than a full image of the star. The low-entropy-origin mechanism survives (a pure seed is a pure seed), but the original narrative must be correspondingly weakened. A future revision should either commit to a branch or, as here, present the fork as the framework's central open parameter.

9 Relation to other frameworks

Conformal cyclic cosmology [29] is the comparison reviewers reach for, and the decisive contrast is informational: CCC *requires* information loss in black-hole evaporation as its entropy-reset mechanism, while Causal Symmetry is defined by its strict denial. The programs are distinguishable in principle — unitarity-restoring structure (echoes, remnants, purity-restoring correlations) favors this framework; confirmed Hawking-point-like anomalies [30, 31] without such structure would favor CCC. Mechanically they also differ: quantum bounce versus conformal rescaling; paired, possibly branching domains versus strictly sequential aeons; local arrows with a time-symmetric meta-structure versus a single arrow threading aeons.

The nearest relatives are instead the **two-arrow cosmologies** — the Janus-point construction [32], spontaneous-inflation pictures [33], the Aguirre–Gratton steady state [34] — and the holographic white-hole origin of [35]. What Causal Symmetry adds to that family is a specific pairing mechanism: censor boundaries and code-subspace transfer, in the language of holographic error correction [14], rather than a statistical origin of the two arrows.

10 Objections and open problems

1. **Firewalls.** The framework needs smooth horizons; the AMPS argument [15] says smoothness, unitarity, and effective field theory conflict. The modern escapes (code subspaces, islands [14, 16]) are structurally friendly to the WHO condition, but compatibility must be shown, not asserted.

2. **The Γ problem** (Section 6): the framework does not yet select among scalings that differ by $e^{-10^{77}}$; WHO-3 is a strong dynamical assumption doing all the observational work in the GW sector.

3. **Branch commitment** (Sections 5, 8): early versus late transition changes both the phenomenology and the unitarity narrative; deciding it is a quantum-gravity calculation.

4. **Operational meaning.** Global conservation across disconnected domains is unobservable from within one domain; the framework's empirical content lives entirely in Sections 6–7, and this should be conceded up front.

5. **White-hole instability.** Long-lived white-hole phases inherit the classical blueshift instability; small holes are argued quantum-stabilized [11].

6. **Energy bookkeeping** — closed by reframing: within one exterior, effective models conserve the ADM mass across the bounce [8]; across disconnected domains, energy balance is not even definable (no shared asymptotic time), and the invariant that crosses the map is information, exactly as Eq. (1) asserts.

11 Conclusion

Causal Symmetry treats black and white holes as dual, information-preserving boundaries: absorption as an isometry, emission as its adjoint on the image, global unitarity as a three-clause boundary condition on Planckian dynamics. The condition is exponentially non-generic ($\bar{F} \sim e^{-S_{BH}}$ for random bounces) — which is precisely what makes it a hypothesis with content — and its observational program is now explicit on both lifetime branches: phase-coherent echo trains with mass-locked delays and combs, or sub-millimeter transients, or Planck-mass remnant dark matter, with the CMB channel bounded below current sensitivity. The framework's fate rests on two calculations it has now specified for the community in which they must be done: the transfer matrix of a spinfoam transition amplitude against the $1/\dim \mathcal{H}_P$ decoupling baseline, and a first-principles $\Gamma(\omega)$. Its cleanest philosophical yield stands regardless: a universe in which every low-entropy beginning is the purified image of another domain's gravitational ending, with nothing ever lost in between.

A Reproducibility

All numerical results are generated by four scripts distributed with the project: `who_fidelity.py` (Haar baseline and tolerance curves, numpy-only Haar sampling via QR of Ginibre matrices), `echo_template.py` (cavity transfer function, waveform, comb), `rw_barrier.py` (Regge–Wheeler scattering; unitarity 6×10^{-13}; reflectivity table `rw_reflectivity.csv`), and the figure scripts of Supplement I. Physical constants: CODATA; $M_{\rm Pl} = 2.176 \times 10^{-8}$ kg, $S_{BH}(M_\odot) = 1.05 \times 10^{77} k_B$.

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